Pre-Calculus Name: Key Date: Period:

Unit 8 Test (ART)

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Target 8.1: I can use and apply fundamental trigonometric identities.

1. Simplify $\sec(-1)\cos(-1)$ to either 1 or -1.

$$\sec(-1)\cos(-1) = \frac{1}{\cos(-1)} \cdot \cos(-1) = \boxed{1}$$

2. Prove $\frac{1}{sin^2x} + \frac{sec^2x}{tan^2x} = 2csc^2x$.

$$\frac{1}{\sin^2 x} + \frac{\sec^2 x}{\tan^2 x} = \frac{1}{\sin^2 x} + \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2}} = \frac{1}{\sin^2 x} + \frac{1}{\cancel{\cos^2 x}} \cdot \frac{\cancel{\cos^2} x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x} + \frac{1}{\sin^2 x}$$

$$= \frac{2}{\sin^2 x}$$

$$= 2\csc^2 x \checkmark$$

3. Simplify the expression $\tan x \cdot \sin x$ to a single trigonometric function.

oops - should have been $\tan x \cdot \cos x$

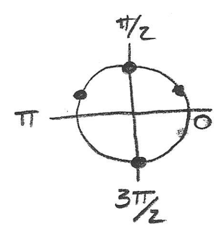
$$\tan x \cdot \cos x = \frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}$$

$$= \boxed{\sin x}$$

4. Find all solutions to the equation in the interval $[0, 2\pi)$ for the equation $2\cos x \sin x - \cos x = 0$.

$$2\cos x \sin x - \cos x = 0$$

$$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$$



$$\cos x(2\sin x - 1) = 0$$

$\cos x = 0$ or $2\sin x - 1 = 0$

$x = \pi/2, 3\pi/2$ $\sin x = \frac{1}{2}$

$x = \pi/6, 5\pi/6$

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Target 8.2: I can prove trigonometric identities.

5. Prove $(\cos x)(\tan x + \sin x \cot x) = \sin x + \cos^2 x$

$$(\cos x)(\tan x + \sin x \cot x) = (\cos x)\left(\frac{\sin x}{\cos x} + \cancel{\sin x}\,\frac{\cos x}{\cancel{\sin x}}\right)$$

$$= \sin x + \cos^2 x \checkmark$$

6. Prove $\frac{\cos^2 x - 1}{\cos x} = -\tan x \cdot \sin x$

$$\frac{\cos^2 x - 1}{\cos x} = \frac{-\sin^2 x}{\cos x}$$

$$= \frac{-\sin x \ \sin x}{\cos}$$

$$= -\tan x \cdot \sin x \checkmark$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$-\sin^2 x = \cos^2 x - 1$

7. Prove $\sec x - \cos x = \sin x \cdot \tan x$

$$\sec x - \cos x = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \frac{\sin x \cdot \sin x}{\cos x}$$

$$= \sin x \cdot \tan x \checkmark$$

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Target 8.3: I can use and apply sum and difference identities.

8. Prove that $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$

$\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} - \sin\frac{\pi}{2}\cos x$

$= \sin x(0) - (1)\cos x$

$= -\cos x$ ✓

9. Evaluate $\cos(75°)$ exactly.

$\cos 75° = \cos(45° + 30°)$

$= \cos 45° \cdot \cos 30° - \sin 45° \cdot \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

10. Write $\sin 37° \cdot \cos 14° + \sin 14° \cdot \cos 37°$ as a single sine or cosine expression.

$\sin(37° + 14°)$

$\boxed{\sin(51°)}$

$\sin x \cdot \cos y + \sin y \cdot \cos x$

$\sin(x + y)$

11. Prove $\sin(x - y) + \sin(x + y) = 2 \sin x \cdot \cos x$ ← oops- should have been y

$\sin(x-y) + \sin(x+y) = (\sin x \cos y - \cancel{\sin y \cos x}) + (\sin x \cos y + \cancel{\sin y \cos x})$

$= \sin x \cos y + \sin x \sin y$

$= 2 \sin x \cos y$ ✓

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Target 8.4: I can use and apply multiple angle identities.

12. Prove $\cos 2x = 2\sin^2 x - 1$. oops - should have been $1 - 2\sin^2 x$

$\cos 2x = \cos(x+x)$

$= \cos x \cos x - \sin x \sin x$

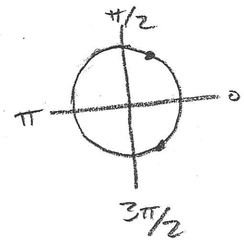
$= \cos^2 x - \sin^2 x$

$= (1-\sin^2 x) - \sin^2 x$

$= 1 - 2\sin^2 x$ ✓

13. Find all solutions to the equation in the interval $[0, 2\pi)$.

$\sin 2x = \sin x$



$\sin 2x = \sin x$

$\sin 2x - \sin x = 0$

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\boxed{0, \pi, \pi/3, 5\pi/3}$

$\sin x = 0$ $\quad$ $2\cos x - 1 = 0$

$\cos x = 1/2$

$x = 0, \pi$ $\quad$ $x = \pi/3, 5\pi/3$

14. Use a half-angle identity to evaluate $\sin\left(\frac{5\pi}{12}\right)$ exactly.

$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$

$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\frac{5\pi}{6}}{2}\right)$

$= \pm\sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}}$

$= \pm\sqrt{\frac{1-(-\sqrt{3}/2)}{2}}$

$= \pm\sqrt{\frac{1+\sqrt{3}/2}{2}}$

$= \pm\sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\pm\sqrt{2+\sqrt{3}}}{2}}$